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A MEASURE OF STRATEGIC STABILITY
BASED ON
TWO-STRIKE NUCLEAR EXCHANGE MODELS

Jeffrey H. Grotte

February 1978

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This paper presents a quantification of the notion of strategic stability as it pertains to changes in US and Soviet nuclear postures. Based on the use of two-strike strategic exchange models, this approach can be used to measure the degree to which proposed posture changes are stabilizing or destabilizing. The categories "predominantly stabilizing" and "predominantly destabilizing" are introduced to describe changes which, in sequence, produce respectively stabilizing and →next page

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FOREWORD

The notion of stability has come to play an important role
in discussions of nuclear armaments. This paper presents a
quantification of the idea of stability as it pertains to changes
in U.S. and Soviet strategic nuclear postures. The concepts of
this paper are based on the use of two-strike strategic exchange
models; in particular, those models that can guarantee optimal
solutions. Such models have only recently been developed. Using
this approach, the degree to which posture changes are stabilizing
or destabilizing can be measured.

A. INTRODUCTION

The notion of stability as it pertains to US and Soviet strategic nuclear postures has become widely accepted. When changes in force structures, readiness, or civil defense planning are proposed, sooner or later they will be labelled either destabilizing by their critics or stabilizing by their proponents. This is not to say that stability is an empty concept to be invoked when desired or denied when convenient, but rather that different analysts can honestly view the same change in different lights. Contributing to this are the following problems:

(1) Stability has not acquired a universally accepted definition.

(2) When discussions of stability are based on mathematical formulations, these formulations are often abstracted to such a degree that it is possible to draw different conclusions from the same data. Such abstraction can occur through aggregation of the data or through compounding simplifying assumptions.

(3) In general, a given posture change will contain both stabilizing and destabilizing elements.

The idea of stability certainly has not been ignored. Indeed there are many notions of stability, some quite well defined, which may apply to questions of nuclear strategy, the arms race, and disarmament. Richardson [11] uses the classical definition of stability of critical points in his ordinary differential equation arms race models. Saaty [12] provides examples of game-theoretic notions of stability and Howard [5] examines a hierarchy of metagame theoretic equilibria. Kahan [6] lists a number of perhaps the most commonly

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accepted characteristics of stabilizing or destabilizing changes--

> Generally speaking, the *stabilizing* category includes relatively invulnerable weapons and programs designed to enhance survivability... [while] systems and strategies geared to negate an opponent's retaliatory capability and having no direct bearing on the preservation of a dependable deterrent... [are] *destabilizing*.

This paper presents an investigation into the question of stability in a two-country context that requires a two-strike nuclear exchange model. This, in itself, it not new; because nuclear confrontation frequently is taken to be a two-sided situation, it is natural to examine strategic questions in terms of possible interactions between the two forces. This often has been done in highly aggregated cases--see for example, McGuire [7], Pitman [10], Richardson [11], and Saaty [12], among others. However, the models described in these papers neglect much of the interaction that results from multiple-weapon-type and warning-time considerations. Recent advances in the state of the art of nonlinear programming have enabled the development of new, optimizing strategic exchange models that can examine some of these considerations to a much greater extent than heretofore possible, yet still provide *globally* optimal solutions. (See Bracken, Falk, Miercort [2]; Bracken, Donelson, Grotte, Marcuse [3]; and Grotte [4].) Combined with the concepts of this paper, these new models permit the development of new analytical tools for analyzing strategic stability.[1] In particular, in some cases these models will provide a response to problem (2) above by allowing complex interactions to be analyzed. Further, by providing a numerical

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[1]Non-globally optimizing two-strike models, such as QUICK [8], VALIMAR [9], and the Arsenal Exchange Model [1], also could be used for the purposes of this paper. Their disadvantage is that they cannot guarantee the optimization of a two-strike objective function; the user will have to carefully select which of the outputs he feels provides a meaningful measure.

measure, we will be able to address problem (3) by determining whether the stabilizing or destabilizing elements of a posture change predominate. In other words, we will be able to discern differences of degree rather than only differences of kind.

Admittedly, our approach has a number of shortcomings. By relying on two-strike models, we maintain a degree of tradition but make no major step towards the solution of problem (1), for some changes that are stabilizing in a two-strike exchange may be destabilizing in a multiple-strike type of exchange (such as the adoption of a "launch on warning" policy). Nonetheless, we feel that the use of globally optimizing two-strike models to provide a numerical measure is valuable and worth discussing.

B. OPTIMIZING TWO-STRIKE MODELS

We will consider two-strike nuclear exchange models of the following form

$$\max_{\underline{x}\varepsilon X} \quad \min_{\underline{y}\varepsilon Y(\underline{x})} \quad f\big(D_R(\underline{y}), D_B(\underline{x})\big)$$

where

X = the set of feasible allocations $\underline{x}$ of strategic weapons belonging to the first striker (for convenience, termed RED).

$Y(\underline{x})$ = the set of feasible retaliations $\underline{y}$ available to the second striker (BLUE) after the attack $\underline{x}$. Thus BLUE may find his retaliation limited by an effective RED counterforce first strike.

$D_B(\underline{x})$ = the damage to BLUE society resulting from attack $\underline{x}$.

$D_R(\underline{y})$ = the damage to RED society resulting from retaliation $\underline{y}$.

$f\big(D_R(\underline{y}), D_B(\underline{x})\big)$ = an objective function that is used to compare

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damage. We require only that f be monotone nonincreasing in $D_R(\underline{y})$ and monotone nondecreasing in $D_B(\underline{x})$.

Measures of damage to society, D_B and D_R, can be found in almost any level of detail. Commonly used comparison functions are

$$D_B(\underline{x}) - D_R(\underline{y}),$$

which is used in IDASNEM [3], and

$$D_B(\underline{x}) \Big/ D_R(\underline{y}).$$

Implicit in the above formulation are a number of assumptions. First, the model is a "first striker's model" in the sense that it is the first striker who makes the first choice. The inner problem—the minimization—represents the first striker's lack of knowledge about the retaliation, and therefore he considers the worst case. Second, the model assumes rationality on the part of both participants, that is, each wishes to influence the objective function in a direction favorable to himself. While this may not be the case in an actual war, it is not an unreasonable assumption in analysis. Third, the model assumes that the war is over after the retaliation, and therefore is unsuitable for the examination of multi-strike exchanges. While this is consistent with many past efforts to analyze strategic postures, future work may rely more on multi-strike analyses. Fourth, the form of the comparison function $f(\cdot,\cdot)$ implies that damage to both sides can be measured in comparable units. Other assumptions surface when one examines the exact form of the functions $D_B(\cdot)$, $D_R(\cdot)$, and the set $Y(\underline{x})$. See, for example, Grotte [4].

The comparison function can also be viewed as a measure of deterrence. As the potential second striker lowers the

comparison function, he reduces the advantages of a first strike, thereby deterring the first striker.

Such models are not easy to solve as they may possess local as well as global optima. Nonetheless, modern algorithms allow us to find the globally optimal solution to many such problems.

C. THE ANALYSIS OF STABILITY

Examining only one side of a nuclear exchange, or even both sides of a single nuclear exchange, is insufficient in analyzing proposed nuclear balance modifications, since each side must consider its own retaliatory capability as well as how its first strike capability will be perceived by the other side. Therefore we must apply any one of the two-strike models twice, with first the US and then Soviet Union designated as first striker. This will yield the two-dimensional vector

$$\underline{m}(p_{US}, p_{SU}) = \left(m_1(p_{US}, p_{SU}), \; m_2(p_{US}, p_{SU}) \right)$$

where $m_1(p_{US}, p_{SU})$ is the value of the model's objective function when the US is designated as first striker; p_{US} is the US nuclear posture (those inputs to the model which describe factors over which the US has control); and p_{SU} is the Soviet nuclear posture (those inputs to the model which describe factors over which the Soviet Union has control). Similarly, $m_2(p_{US}, p_{SU})$ is the optimal objective function value when the Soviet Union is designated first striker.

Changing either or both of the US or Soviet postures will result in a change in the vector $\underline{m}$. Let us examine how $\underline{m}$ might change. Consider Figure 1.

First suppose the US unilaterally alters its posture. The US first strike capability follows the m_1 axis which represents

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m_2

b

II   I

SU

First

Strike

(US   Objective   c ————————— $\underline{m}$ ————————— a

retaliation) Function

III   IV

d

m_1

US First Strike Objective Function ⟶

⟵ (Soviet retaliation)
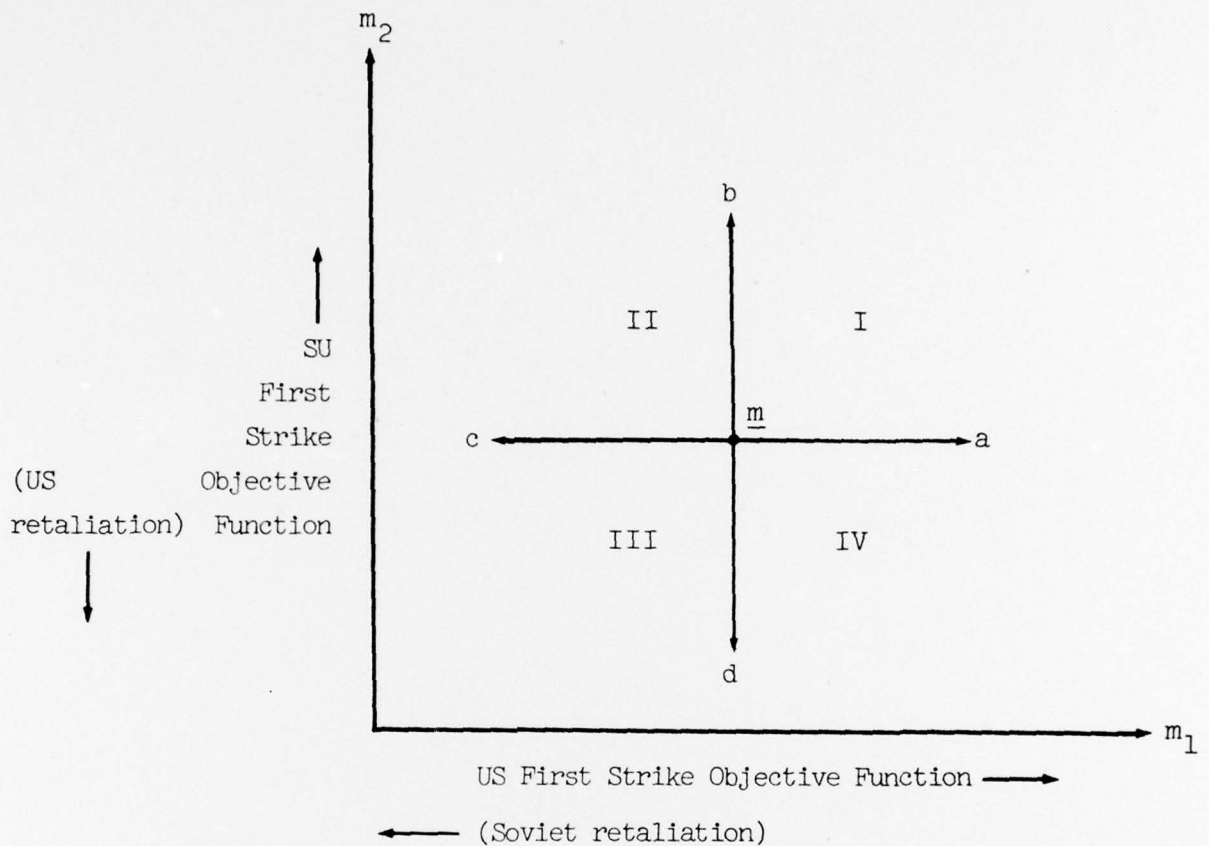
Figure 1. STABILIZING AND DESTABILIZING REGIONS

the value of the model objective function when the US is the
first striker (if damage difference were used, this would be
damage to the Soviet Union minus damage to the US). The US
retaliatory capability is displayed on the m_2 axis which is the
objective function value when the Soviet Union is the first
striker (in the damage difference case, this is damage to the
US minus damage to the Soviet Union). The arrows by the axes
indicate the directions which reflect improvement. If the
posture changes are such that $\underline{m}$ moves in direction a, the US
has improved its first strike capability but has not changed
its retaliatory capability at all; this commonly would be
considered a destabilizing change. If $\underline{m}$ moves in direction b,

the US has *degraded* its retaliatory capability while leaving its first strike capability unchanged. Such a posture change on the part of the US is unlikely and also would be considered destabilizing. If $\underline{m}$ moves in direction c, the US first strike capability has been degraded while the retaliatory capability remains unchanged. This generally would be considered a stabilizing change. In direction d, the US has improved its retaliatory capability while not altering its first strike capability; again, this would be considered a stabilizing change.

These directions define four regions depicted in Figure 1. From the above discussion, it is evident that region I is a region of destabilizing US changes, since it comprises those changes which improve the US first strike while degrading the US retaliation; region II is an unlikely region for a US move; region III is a region of stabilizing US posture change; and and region IV is an ambiguous region. Most unitaleral US posture changes will move $\underline{m}$ into region IV because force posture changes generally result in improved first strike capability, as well as a more secure retaliatory force.

Of course, one must examine the change to $\underline{m}$ when the Soviet Union makes a posture change. The same discussion as the above holds except that the reader should switch references to b and a, and c and d, and regions II and IV. Thus while region III remains a region of stabilizing changes, most Soviet changes can be expected to result in $\underline{m}$ moving into region II. According to this simple approach, US and Soviet posture changes will be opposed in the sense of moving $\underline{m}$ in more or less opposite directions.

The fact that US posture changes will tend to move $\underline{m}$ into region IV while Soviet posture changes will move $\underline{m}$ into region II is also a restatement of problem (3) of Section A--posture changes have both stabilizing and destabilizing elements.

Let us examine these two regions in somewhat more detail. Consider Figure 2. Observe the broken line $\overline{pq}$ that bisects regions II and IV. A US posture change that moves $\underline{m}$ into area x will be both stabilizing and destabilizing, but the destabilizing element will be relatively greater. Thus we will call any US posture change that moves $\underline{m}$ into x *predominantly destabilizing*. On the other hand, any US posture change that moves $\underline{m}$ into area y will be called *predominantly stabilizing*. Similarly, area z is the area of predominantly destabilizing Soviet posture changes, while area w is the area of predominantly stabilizing Soviet posture changes. This distinction is important because if a predominantly stabilizing move on one side is followed by a predominantly stabilizing reaction on the other, the net result can well be stabilizing; whereas a predominantly destabiling move followed by a predominantly destabilizing reaction will produce a net result that is at best predominantly destabilizing and which may be in region I, the totally destabilizing region.

Consider the example in Figure 3. Suppose the present US and Soviet postures correspond to a vector $\underline{m}^1$. Assume the Soviets change their posture in a predominantly stable fashion in a move to $\underline{m}^2$, and that the US responds with a predominantly stable change leading to the final result $\underline{m}^3$ in region III. This posture change and reaction results in a more stable balance, even though both posture changes have destabilizing elements.

It is true, of course, that a pair of predominantly stabilizing posture changes might end up in either of the ambiguous regions II or IV, but even so, at worst the result will fall in a predominantly stabilizing area.

Figure 4 shows how two predominantly destabilizing moves can produce a totally destabilizing result. In this case, the predominantly destabilizing Soviet change results in $\underline{m}^2$. A predominantly destabilizing US response produces the net result
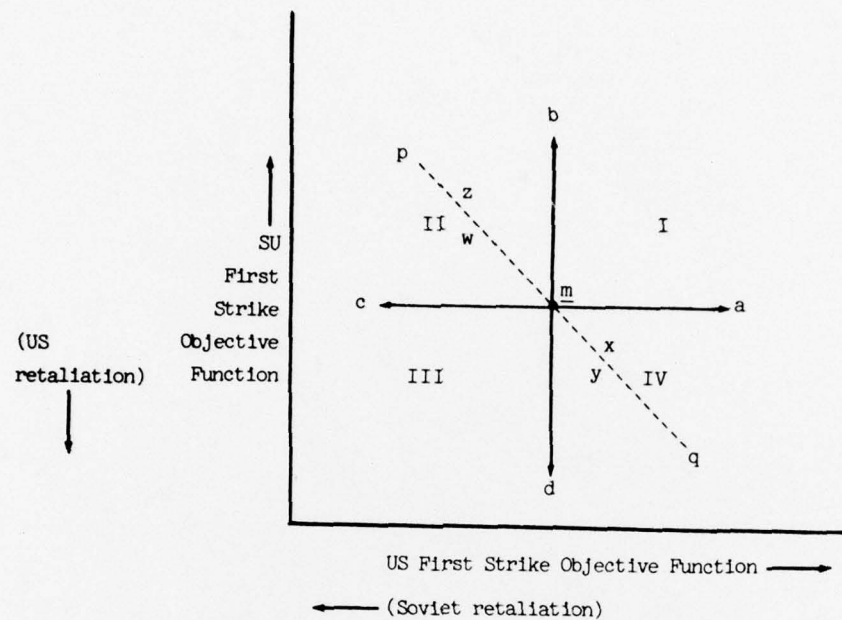
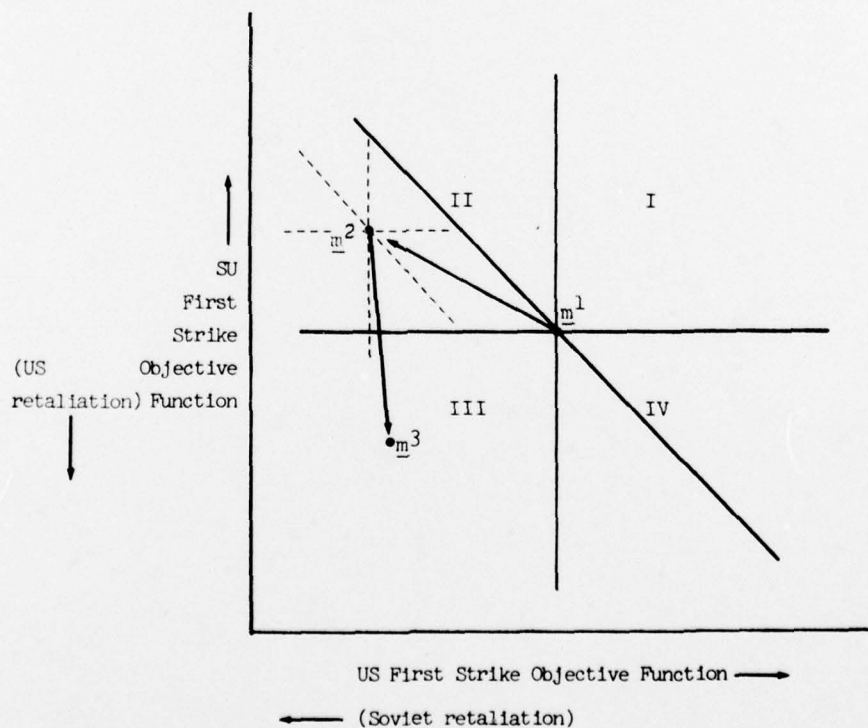Figure 2. PREDOMINANTLY STABILIZING AND DESTABILIZING AREAS

Figure 3. TWO PREDOMINANTLY STABILIZING MOVES RESULT IN A
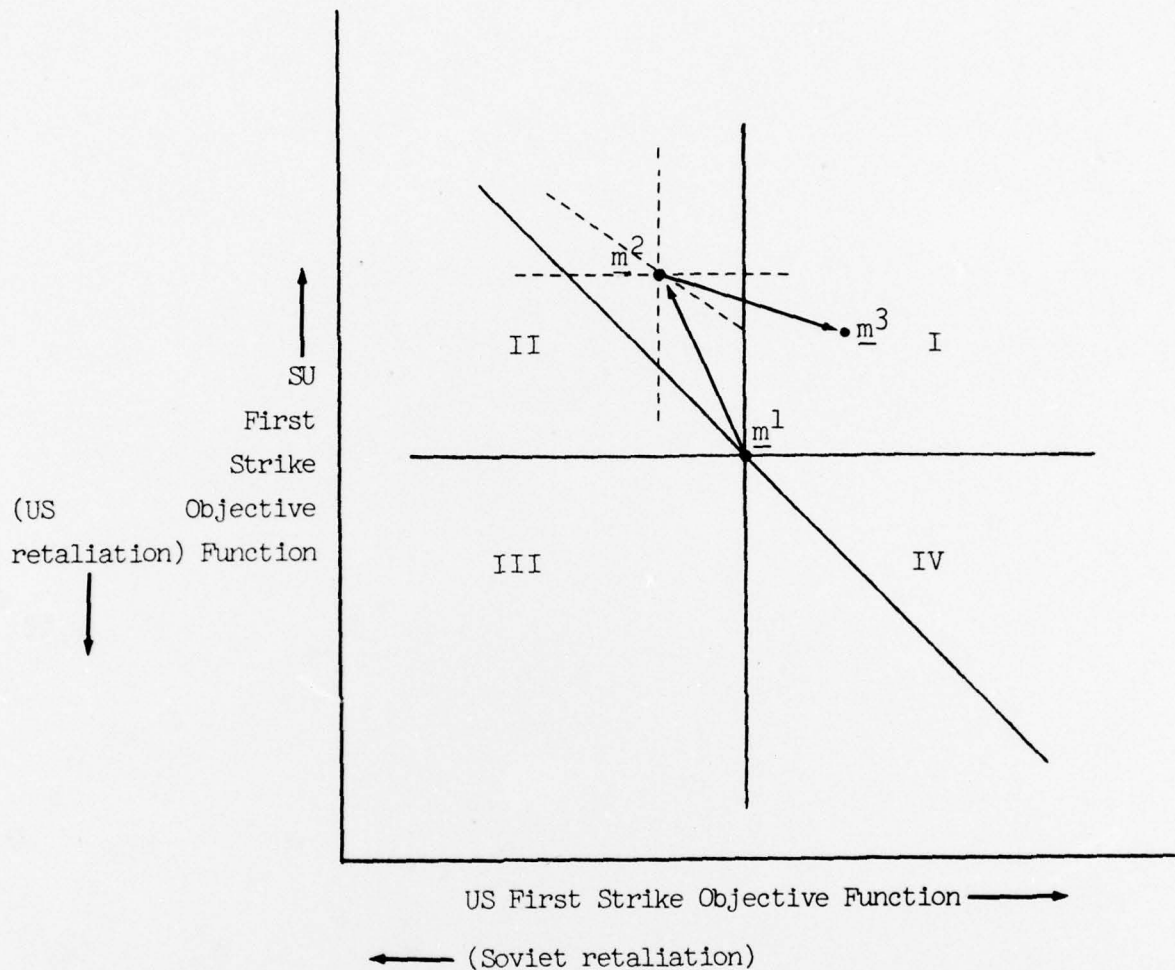NET TOTALLY STABILIZING MOVE

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Figure 4. TWO PREDOMINANTLY DESTABILIZING
MOVES RESULT IN A NET TOTALLY
DESTABILIZING MOVE

$\underline{m}^3$ which lies in totally destabilizing region I. It is signifi-
cant that even if the Soviet Union chooses a destabilizing or a
predominantly destabilizing posture change, it is still to the
advantage of the US to follow with a stabilizing or a predominantly
stabilizing response. Figure 5 depicts the same predominantly
destabilizing Soviet move as in Figure 4, but, when followed by
a predominantly stabilizing response by the US, the end result
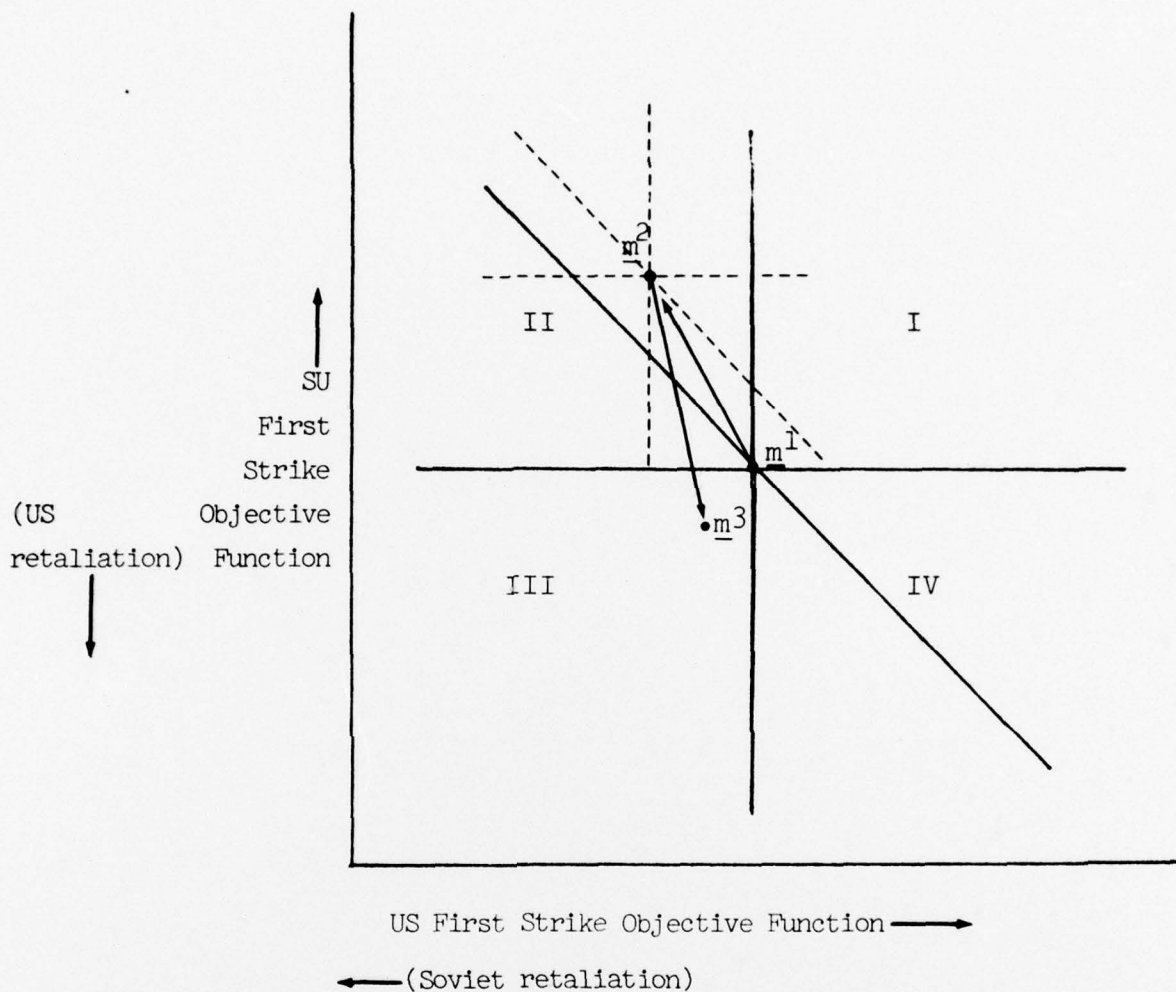$\underline{m}^3$ is in the totally stabilizing region III.

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Figure 5. A PREDOMINANTLY DESTABILIZING MOVE WITH A
PREDOMINANTLY STABILIZING RESPONSE RESULTS
IN A TOTALLY STABILIZING MOVE

D.   ON ARMS AGREEMENTS

Using more or less the same procedures as above, we also
have a means of examining the stabilizing or destabilizing
effects of arms agreement. An arms agreement essentially re-
stricts each side to sets of feasible postures P_{US} and P_{SU} from

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which each side chooses one posture (P_{US} and P_{SU}).[1] Theoretically, at least, the set of possible outcomes of an arms agreement

$$M(P_{US}, P_{SU}) = \{\underline{m}(p_{US}, p_{SU}) \mid p_{US} \in P_{US},\ p_{SU} \in P_{SU}\}.$$

can be examined.

Since one purpose of an agreement is to reach a stable pair of postures, an agreement will be successful if $M(P_{US}, P_{SU})$ lies entirely within region III, relative to the present $\underline{m}$. If $M(P_{US}, P_{SU})$ intersects region II, it will be unacceptable to the US. If it intersects region IV, it will be unacceptable to the Soviet Union, whereas if it intersects region I, it could be worse than no agreement, provided both sides restricted themselves to predominantly stable changes. For example, suppose the present postures correspond to $\underline{m}$ and three arms agreements are under consideration, corresponding to feasible posture sets (P'_{US}, P'_{SU}), $(\hat{P}_{US}, \hat{P}_{SU})$, and $(\tilde{P}_{US}, \tilde{P}_{SU})$. Suppose the M-sets for these posture sets are as in Figure 6. Only (P'_{US}, P'_{SU}) is clearly acceptable.

CONCLUSION

There is no doubt that deterrence is of crucial importance to both the US and the Soviet Union. It is also true that each side, for the most part, upgrades its strategic posture on the basis of unilateral considerations and that therefore there is always the possibility that deterrence will be upset, or destabilized, to the benefit of neither party. It follows that one of these unilateral considerations should be the extent to which a posture change affects the "nuclear balance," and it is indeed

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[1] Perhaps one could treat this as a two-stage game wherein the first stage consists of negotiating the posture sets P_{US}, P_{SU} in a cooperative fashion, and the second stage is a zero-sum game with the pure strategies having been determined by the outcome of the first stage.
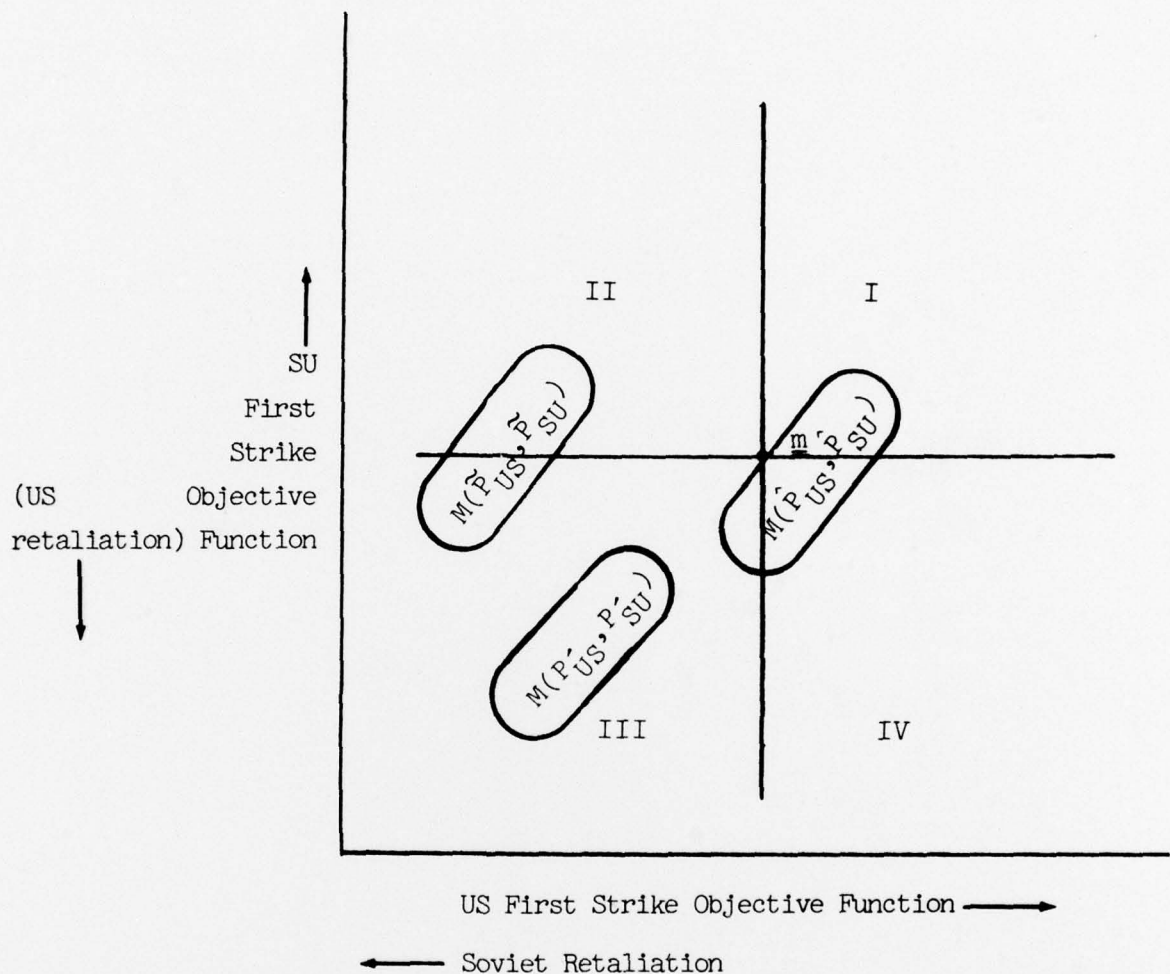
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Figure 6. POSSIBLE ARMS AGREEMENTS

true that stability-destability questions have been raised about
various posture changes in the past, at least in the US. What
we hope to contribute in this paper is a technique employing
quantitative analytic methods (two-strike modeling) to look at
the stability-destability question (in the massive attack-massive
retaliation scenario) as a continuum of possibilities rather
than as an either/or proposition. We also introduce the notions
of predominantly stabilizing and predominantly destabilizing
changes which, in combinations, produce quite different end
results.

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